CS457 Assignment 1

Cedric Hölzl

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1 Problem 1.3.1

Is the area centroid of a quadrilateral equal to the average of its vertices?

Assume we have a quadrilateral, with vertices ABCD:

$$(A_x, A_y), (B_x, B_y), (C_x, C_y), (D_x, D_y)$$

We can compute the average of its vertices as follows:

$$Avg_x : \frac{(A_x + B_x + C_x + D_x)}{4}$$
$$Avg_y : \frac{(A_y + B_y + Cy + D_y)}{4}$$

We now compute the area centroid. For this we can simplify the problem by computing the centroid of two triangles (created by the diagonal BC):
Triangle ABC:

$$Trig_x^{ABC} : \frac{(A_x + B_x + C_x)}{3}$$
$$Trig_y^{ABC} : \frac{(A_y + B_y + C_y)}{3}$$
$$Trig_{Area}^{ABC} : \frac{(A_xB_y - B_xA_y + B_xC_y - C_xB_y + C_xA_y - A_xC_y)}{2}$$

Triangle BCD:

$$
\begin{aligned}
Trig_x^{BCD} :& \frac{(B_x + C_x + D_x)}{3} \\
Trig_y^{BCD} :& \frac{(B_y + C_y + D_y)}{3} \\
Trig_{Area}^{BCD} :& \frac{(D_x B_y - B_x D_y + B_x C_y - C_x B_y + C_x D_y - D_x C_y)}{2}
\end{aligned}
$$

We can then combine them to have the "overall centroid":

$$
\begin{aligned}
Cent_x :& \frac{Trig_x^{ABC} Trig_{Area}^{ABC} + Trig_x^{BCD} Trig_{Area}^{BCD}}{Trig_{Area}^{ABC} + Trig_{Area}^{BCD}} \\
Cent_y :& \frac{Trig_y^{ABC} Trig_{Area}^{ABC} + Trig_y^{BCD} Trig_{Area}^{BCD}}{Trig_{Area}^{ABC} + Trig_{Area}^{BCD}}
\end{aligned}
$$

We can now verify if $Cent_x = Avg_x$ or $Cent_y = Avg_y$:

$$
\begin{aligned}
\frac{Trig_x^{ABC} Trig_{Area}^{ABC} + Trig_x^{BCD} Trig_{Area}^{BCD}}{Trig_{Area}^{ABC} + Trig_{Area}^{BCD}} = Avg_x \\
\frac{Trig_y^{ABC} Trig_{Area}^{ABC} + Trig_y^{BCD} Trig_{Area}^{BCD}}{Trig_{Area}^{ABC} + Trig_{Area}^{BCD}} = Avg_y
\end{aligned}
$$

By substituting the variables above, we obtain a complex formula that when trying to simplify we can be certain not being equivalent. To be certain we can verify it with a counter example.
Let the following be our quadrilateral:

$$
\begin{aligned}
A =& (0, 0) \\
B =& (0.7, 0.5) \\
C =& (1, 0) \\
D =& (1, 1)
\end{aligned}
$$

We then obtain the following:

$$
\begin{aligned}
Avg =& (\frac{2.7}{4}, \frac{1.5}{4}) \\
Trig^{ABC} =& (\frac{1.7}{3}, \frac{0.5}{3}) \\
Trig^{ABC}_{Area} =& \frac{0.5}{2} \\
Trig^{BCD} =& (\frac{2.7}{3}, \frac{1.5}{3}) \\
Trig^{BCD}_{Area} =& \frac{0.3}{2} \\
Cent_x =& \frac{\frac{1.7}{3}\frac{0.5}{2} + \frac{2.7}{3}\frac{0.3}{2}}{\frac{0.5}{2} + \frac{0.3}{2}} = \frac{\frac{0.85+0.81}{6}}{0.4} = \frac{\frac{0.83}{3}}{0.4} = \frac{2.766...}{4} \\
Cent_y =& \frac{\frac{0.5}{3}\frac{0.5}{2} + \frac{1.5}{3}\frac{0.3}{2}}{\frac{0.5}{2} + \frac{0.3}{2}} = \frac{\frac{0.25+0.45}{6}}{0.4} = \frac{\frac{35}{3}}{0.4} = \frac{1.166...}{4}
\end{aligned}
$$

We can clearly see that $\frac{2.766...}{4} \neq \frac{2.7}{4}$ and $\frac{1.166...}{4} \neq \frac{1.5}{4}$. This proves that they are infact not the same thing for such a quadrilateral. This might hold true however for certain quadrilaterals such as a square,...