

Registers

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Register

- A *register* has two operations: *read()* and *write()*
- Sequential specification
- • *read()*
 - return(x)
- • *write(v)*
 - $x \leftarrow v$; return(ok)

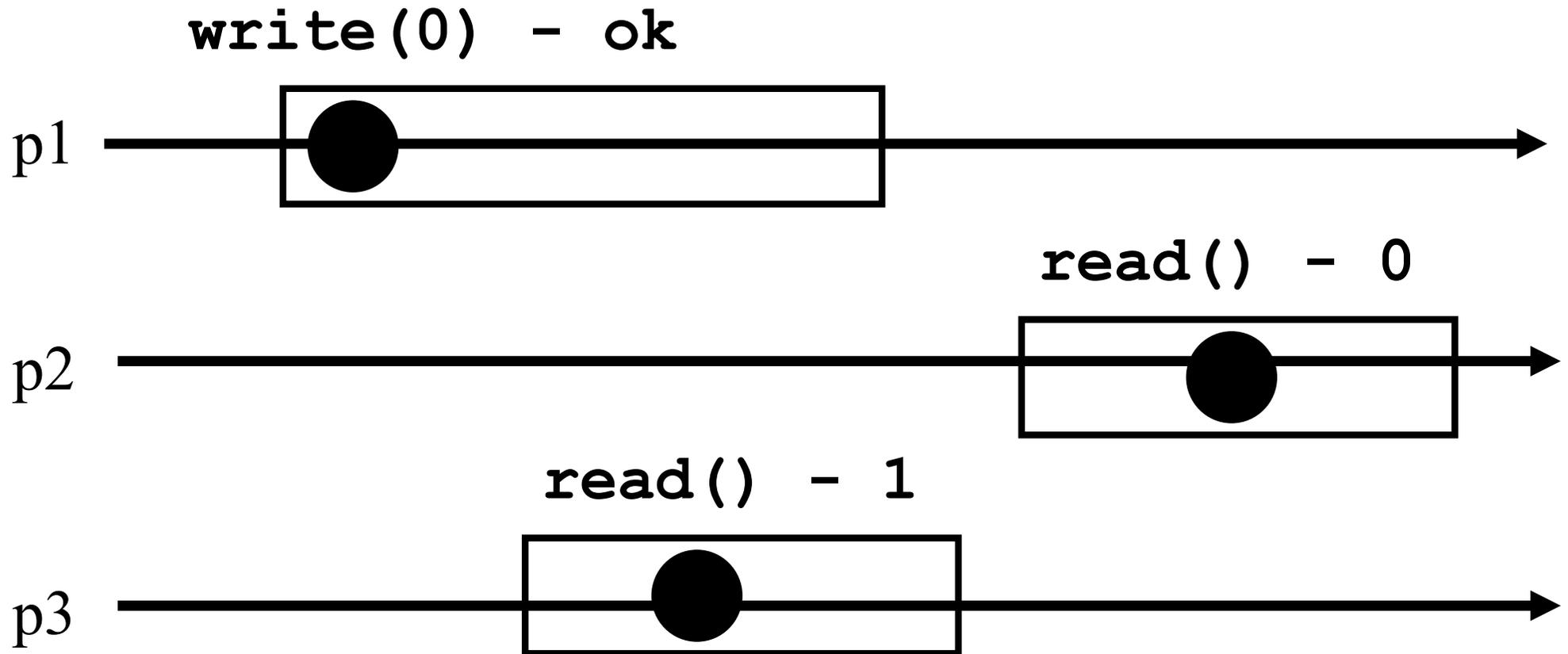
Simplifications

- We assume that *registers* contain only integers
- Unless explicitly stated otherwise, *registers* are initially supposed to contain 0

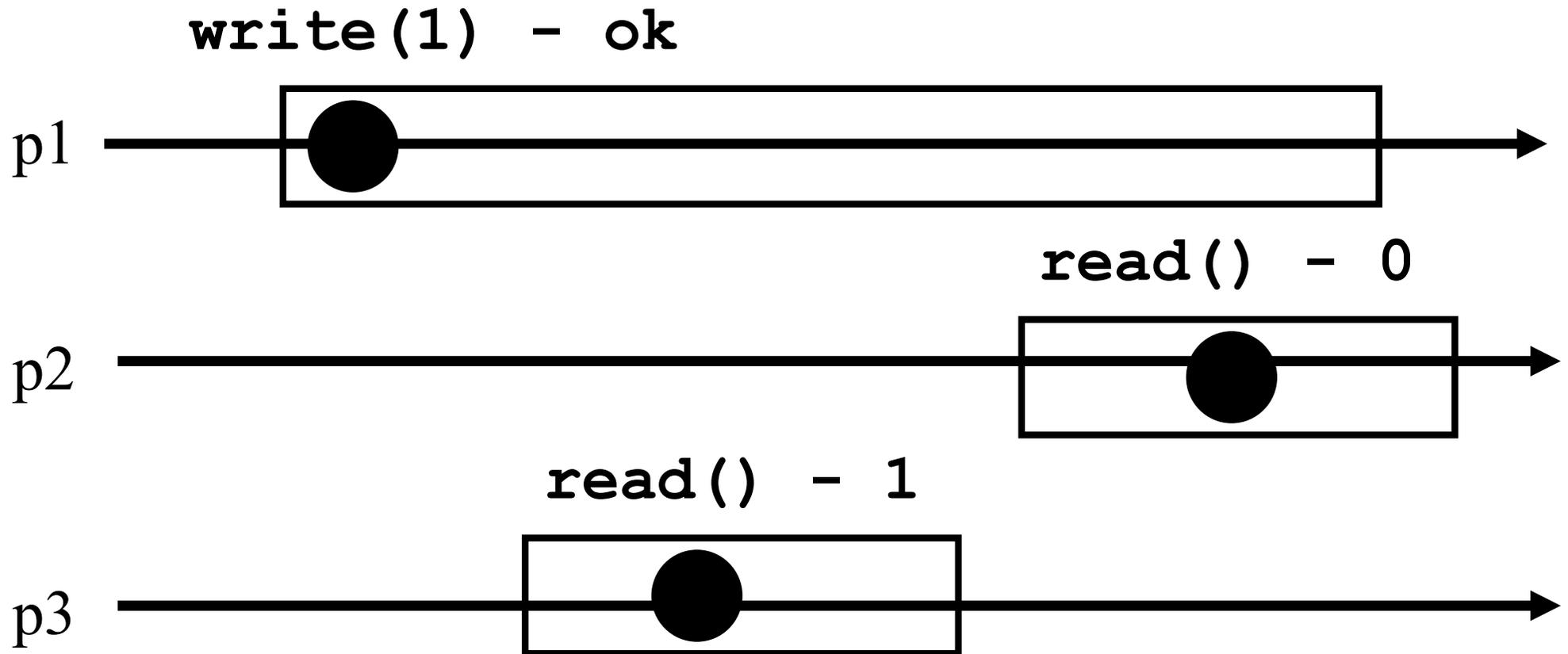
Dimensions

- Dimension 1: binary (boolean) – multivalued
- Dimension 2:
 - SRSW (single reader, single writer)
 - MRSW (multiple reader, single writer)
 - MRMW (multiple reader, multiple writer)
- Dimension 3: safe – regular – atomic

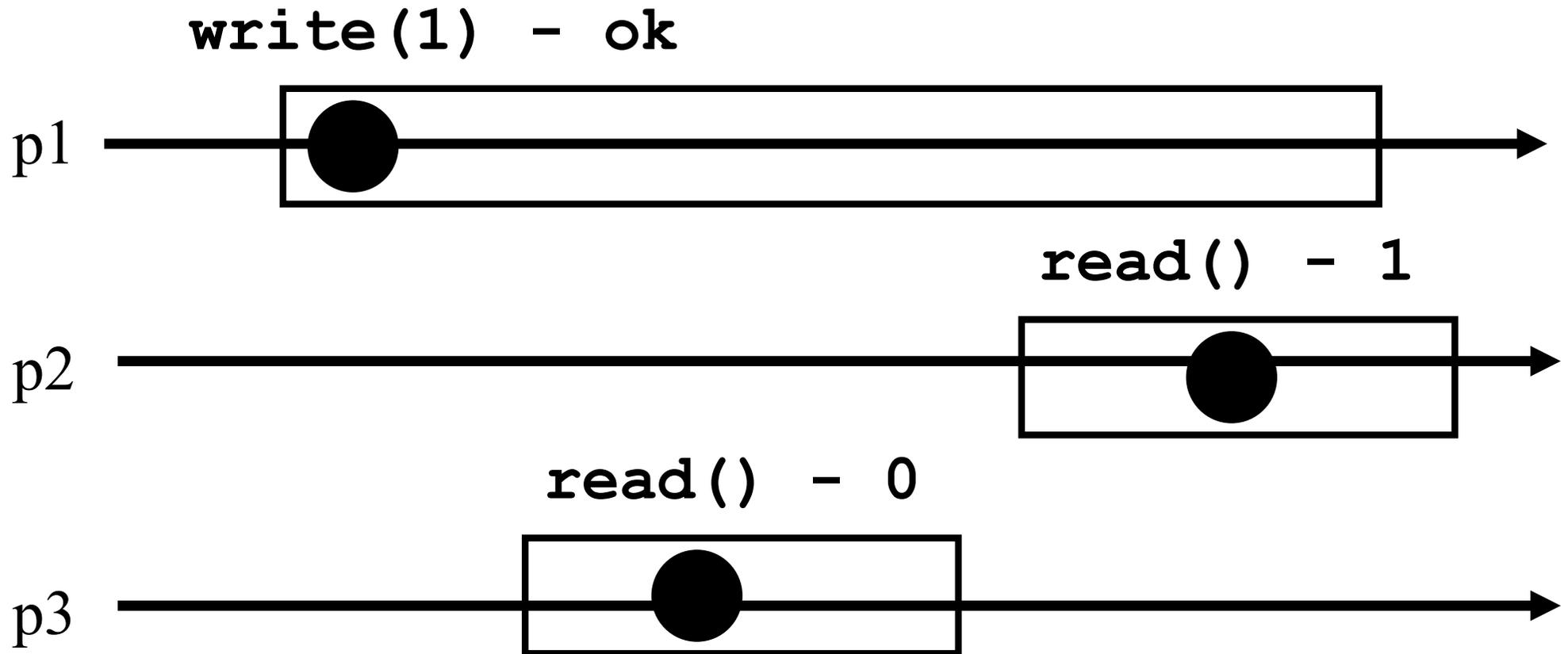
Safe execution



Regular execution



Atomic execution



2 decades of hard work

- Theorem: A multivalued MRMW atomic *register* can be implemented with binary SRSW safe *registers*

Conventions (1)

- The process executing the code is implicitly assumed to be *pi*
- We assume a system of *n* processes
- NB. We distinguish base and high-level registers

Conventions (2)

- The operations to be implemented are denoted ***Read()*** and ***Write()***
- Those of the base registers are denoted ***read()*** and ***write()***
- We omit the ***return(ok)*** instruction at the end of ***Write()*** implementations

(1) From (binary) SRSW safe to (binary) MRSW safe

- ☛ We use an array of SRSW *registers*

Reg[1,...,N]

- ☛ **Read()**

- ☛ return (Reg[i].read());

- ☛ **Write(v)**

- ☛ for j = 1 to N

- ☛ Reg[j].write(v);

From (binary) SRSW safe to (binary) MRSW safe

- The transformation works also for multi-valued registers and regular ones
- It does not however work for atomic registers

(2) From binary MRSW safe to binary MRSW regular

- We use one MRSW safe register

- **Read()**

- `return(Reg.read());`

- **Write(v)**

- if `old ≠ v` then

- `Reg.write(v);`

- `old := v;`

From binary MRSW safe to binary MRSW regular

- The transformation works for single reader ***registers***
- It does not work for multi-valued ***registers***
- It does not work for atomic ***registers***

(3) From binary to M-valued MRSW regular

- We use an array of MRSW registers
Reg[0,1,...,M] init to [1,0,...,0]
- **Read()**
 - for j = 0 to M
 - if Reg[j].read() = 1 then return(j)
- **Write(v)**
 - Reg[v].write(1);
 - for j = v-1 downto 0
 - Reg[j].write(0);

From binary to M-valued MRSW regular

- The transformation would not work if the Write() would first write 0s and then 1
- The transformation works for regular but NOT for atomic registers

(4) From SRSW regular to SRSW atomic

- We use one SRSW register Reg and two local variables t and x
- **Read()**
 - $(t',x') = \text{Reg.read}();$
 - if $t' > t$ then $t := t'; x := x';$
 - return(x)
- **Write(v)**
 - $t := t + 1;$
 - $\text{Reg.write}(v,t);$

From SRSW regular to SRSW atomic

- The transformation would not work for multiple readers
- The transformation would not work without timestamps
(variable t represents logical time, i.e., timestamp)

(5) From SRSW atomic to MRSW atomic

- We use $N \times N$ SRSW atomic registers $RReg[(1,1),(1,2),\dots,(k,j),\dots,(N,N)]$ to communicate among the readers
 - In $RReg[(k,j)]$ the reader is p_k and the writer is p_j
- We also use N SRSW atomic ***registers*** $WReg[1,\dots,N]$ to store new values
 - the writer in all these is p_1
 - the reader in $WReg[k]$ is p_k

(5) From SRSW atomic to MRSW atomic (cont'd)

- **Write(v)**
 - `t1 := t1+1;`
 - for `j = 1` to `N`
 - `WReg.write(v,t1);`

(5) From SRSW atomic to MRSW atomic (cont'd)

Read()

- for $j = 1$ to N do
 - $(t[j], x[j]) = \text{RReg}[i, j].\text{read}();$
- $(t[0], x[0]) = \text{WReg}[i].\text{read}();$
- $(t, x) := \text{highest}(t[..], x[..]);$
- for $j = 1$ to N do
 - $\text{RReg}[j, i].\text{write}(t, x);$
- return(x)

Value with highest timestamp

From SRSW atomic to MRSW atomic

- The transformation would not work for multiple writers
- The transformation would not work if the readers do not communicate (i.e., if a reader does not write)

(6) From MRSW atomic to MRMW atomic

- We use N MRSW atomic registers $\text{Reg}[1,\dots,N]$; the writer of $\text{Reg}[j]$ is p_j
- **Write(v)**
 - for $j = 1$ to N do
 - $(t[j],x[j]) = \text{Reg}[j].\text{read}();$
 - $(t,x) := \text{highest}(t[..],x[..]);$
 - $t := t+1;$
 - $\text{Reg}[i].\text{write}(t,v);$

(6) From MRSW atomic to MRMW atomic (cont'd)

Read()

- for $j = 1$ to N do
 - $(t[j], x[j]) = \text{Reg}[j].\text{read}();$
- $(t, x) := \text{highest}(t[..], x[..]);$
- return(x)